

ANOMALOUS DIFFUSION AND NONLINEAR RELAXATION PHENOMENA IN STOCHASTIC MODELS OF INTERDISCIPLINARY PHYSICS

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- The investigation of steady-state probability, spectral-correlation characteristics of anomalous diffusion in the form of Lévy flights and transient dynamics of nonlinear systems characterized by confined potentials.
- The study of the stochastic dynamic of the resistive switching mechanism in the memristive systems.

Statistical characteristics of diffusion

Introduction

To explore Lévy flights we have to consider Langevin equation with Lévy stable noise

$$\frac{dx}{dt} = -\frac{dU}{dx} + \xi_{\alpha}(t)$$

and correspondingly the fractional Fokker-Planck equation

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial x} \left[U'(x) P \right] + D_{\alpha} \frac{\partial^{\alpha} P}{\partial |x|^{\alpha}}$$

with the fractional space derivative

$$D_{\alpha}\frac{\partial^{\alpha}P(x,t)}{\partial|x|^{\alpha}} = Q_{\alpha}\int_{-\infty}^{+\infty}\frac{P(x-z,t) - P(x,t)}{|z|^{1+\alpha}}dz. \qquad Q_{\alpha} = \frac{D_{\alpha}\Gamma(\alpha+1)\sin\left(\pi\alpha/2\right)}{\pi}.$$

In general one well potentials of the type

$$U(x) = \frac{|x|^c}{c}$$

the variance is finite only if $c>4-\alpha$, that is the potential wall is "steep enough".

The residence time $T(x_0)$ in the given domain G for the infinite observation time reads

$$T(x_0) = \int_0^\infty \mathbb{1}_G (x(t)) dt, \qquad \mathbb{1}_G(y) = \begin{cases} 1, & y \in G, \\ 0, & \text{otherwise.} \end{cases}$$

The mean residence time in the domain is

$$\langle T(x_0) \rangle = \int_0^\infty Pr(t, x_0) dt = \int_0^\infty dt \int_G P(x, t | x_0, 0) dx$$

For parabolic potential we arrive at the following linear differential equation

$$\frac{dx}{dt} = bx + \xi_{\alpha}(t)$$

with the solution

$$x(t) = x_0 e^{bt} + \int_0^t e^{b(t-\tau)} \xi_\alpha(\tau) d\tau$$



The unstable parabolic potential

$$U(x) = -bx^2/2 \ (b > 0)$$

$$\vartheta(k,t) = \left\langle e^{ikx(t)} \right\rangle \Longrightarrow P(x,t|x_0,0) \Longrightarrow \Pr(t,x_0)$$

NES effect for Lévy flights in inverse parabolic potential



The exact formula for the average residence time as a function of the initial conditions, the parameters of the system and of Lévy noise source

$$\langle T(x_0) \rangle = \frac{2}{\pi b} \int_0^\infty \frac{\sin kL}{k} \exp\left\{\frac{(\sigma k)^\alpha}{\alpha b}\right\} dk$$
$$\times \int_k^\infty \frac{\cos (qx_0)}{q} \exp\left\{-\frac{(\sigma q)^\alpha}{\alpha b}\right\} dq$$

We examine the steady-state Lévy flights in the asymmetric bistable quartic potential

$$U(x) = \gamma \left(\frac{x^4}{4} - \frac{ax^2}{2} - bx\right)$$

where γ is the potential steepness and b is the asymmetry parameter (a, b, $\gamma > 0$).

The steady-state regime

It is more convenient to find the steady-state characteristic function firstly.

- apply Fourier transformation to the stationary Fokker-Planck equation;
- substitute the potential form;
- consider the case of Cauchy-stable noise (Lévy index $\alpha = 1$).

$$\frac{d^3\vartheta_{st}}{dk^3} + a\frac{d\vartheta_{st}}{dk} + [ib - \beta_1 \operatorname{sgn}(k)] \vartheta_{st} = 0.$$

 $\vartheta_{st}(k) = Ce^{zk}$ x > 0 $z^3 + az + (ib - \beta_1) = 0.$ $z_k = -x_k + iy_k$ $(x_k > 0)$

The values of parameters reads as

$$a_{1} = \frac{x_{2}}{x_{1} + x_{2}} \left[1 - \frac{2x_{1} (x_{1} - x_{2})}{(y_{2} - y_{1})^{2} + (x_{2} - x_{1})^{2}} \right], \qquad b_{1} = \frac{2x_{1}x_{2}(y_{2} - y_{1})}{(x_{1} + x_{2}) \left[(y_{2} - y_{1})^{2} + (x_{2} - x_{1})^{2}\right]}, \\ a_{2} = 1 - a_{1}, \qquad b_{2} = -b_{1}$$

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The corresponding steady-state probability distribution can be found as

$$P_{st}(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \vartheta_{st}(k) e^{-ikx} dk = \frac{1}{\pi} \left\{ \frac{b_1(x-y_1) + a_1x_1}{(x-y_1)^2 + x_1^2} + \frac{b_2(x-y_2) + a_2x_2}{(x-y_2)^2 + x_2^2} \right\}$$



Fig. 1. Asymmetric bistable quartic potential (red curve) and stationary PDF of the particle position for different values of the noise intensity parameter *D*₁. The values of parameters: a=7, b=6.

Correlation time in symmetric bistable potential

The correlation time for discontinuous Markovian process x(t)

$$\tau_{c} = \frac{\varphi'\left(0\right)}{\langle x, x \rangle_{st}}$$

Symmetric bistable quartic potential (*b*=0) $U(x) = \Delta U \left(1 - \frac{x^2}{a^2}\right)^2$

The exact analytical formula for the correlation time of confined Cauchy-Lévy flights

$$\tau_{c} = \frac{2}{\sqrt{3}\gamma(p^{2} - q^{2})} \operatorname{arctg}\left(\frac{1}{\sqrt{3}}\frac{p+q}{p-q}\right) \qquad p = \left(\sqrt{\left(\frac{a^{2}}{3}\right)^{3} + \left(\frac{D_{1}}{2\gamma}\right)^{2}} + \frac{D_{1}}{2\gamma}\right)^{1/3}, \\ q = \left(\sqrt{\left(\frac{a^{2}}{3}\right)^{3} + \left(\frac{D_{1}}{2\gamma}\right)^{2}} - \frac{D_{1}}{2\gamma}\right)^{1/3}, \quad (p > q).$$

Comparison with the result for monostable symmetric quartic potential:

$$a = 0 \qquad \longrightarrow \qquad \tau_c = \frac{\pi}{3\sqrt{3}\sqrt[3]{\gamma D^2}}$$

(A.A.Dubkov,B,Spagnolo,Eur.Phys.J.,216,31-35 (2013)) ⁹

Correlation time in symmetric bistable potential



The dependence of the correlation time on the height of the potential barrier ΔU at fixed positions of the potential well (*a*=1) for different values of the noise intensity parameter D_1 .

For a sufficiently high potential barrier (or small noise intensity parameter D_1):

$$\Delta U \over a D_1 \gg 1$$
 \longrightarrow $au_c \simeq {\pi a \over 2 D_1}$ does no height o

does not depend on the height of potential barrier!!!

Correlation time in symmetric bistable potential



The correlation time versus the position of the potential wells in the normal (left) and log-log scale (right) at the fixed height of the potential barrier $\Delta U=0.1$ for different values of the noise intensity parameter D_1 .

For a sufficiently large
$$a$$
 $a \gg \frac{\Delta U}{D_1} \longrightarrow \tau_c \simeq \frac{\pi}{3\sqrt{3}} \sqrt[3]{\frac{a^4}{4\Delta U D_1^2}} \sim a^{4/3}$

The general expressions for stationary probability distribution in the case of the symmetric steep potential well $U(x) = \frac{\gamma}{2m} \left(\frac{x}{L}\right)^{2m}$ for the anomalous diffusion in the form of Lévy flights with index Lévy $\alpha = 1$ have the following form

$$P_{st}(x) = \frac{\beta^{4n+1}}{\pi (x^2 + \beta^2)} \prod_{l=0}^{n-1} \frac{1}{x^4 - 2\beta^2 x^2 \cos\left[\pi (4l+1)/(4n+1)\right] + \beta^4} \qquad \mathbf{m} = 2\mathbf{n} + 1$$

$$P_{st}(x) = \frac{\beta^{4n-1}}{\pi} \prod_{l=0}^{n-1} \frac{1}{x^4 - 2\beta^2 x^2 \cos\left[\pi (4l+1)/(4n-1)\right] + \beta^4} \qquad \mathbf{m} = 2\mathbf{n}$$

A.A. Dubkov, B.Spagnolo, Acta Phys. Pol. B 38,1745 (2007)

$$P_{st}(x) = \begin{cases} \frac{1}{\pi\beta} \exp\left\{\sum_{k=1}^{\infty} \frac{1}{2\cos\frac{\pi k}{2m-1}} \cdot \frac{1}{k} \left(\frac{x}{\beta}\right)^{2k}\right\}, & |x| < \beta;\\ \frac{1}{\pi\beta} \left(\frac{\beta}{x}\right)^{2m} \exp\left\{\sum_{k=1}^{\infty} \frac{1}{2\cos\frac{\pi k}{2m-1}} \cdot \frac{1}{k} \left(\frac{\beta}{x}\right)^{2k}\right\}, & |x| > \beta. \end{cases} \qquad \beta = L^{\frac{2m-1}{2}} \sqrt{DL/\gamma}.$$

The smooth symmetrical steep potential

The infinitely deep rectangular potential well

L.

L

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$$U(x) = \frac{\gamma}{2m} \left(\frac{x}{L}\right)^{2m} \qquad \xrightarrow{\text{ in the limit } \mathbf{m} \to \infty} \qquad U(x) = \begin{cases} 0, & |x| < \infty, \\ \infty, & |x| > 0 \end{cases}$$

The steady-state probability distribution in Cauchy case $\alpha = 1$ has the form of well-known distribution of arcsine

$$P_{st}(x) = \begin{cases} \frac{1}{\pi} \frac{1}{\sqrt{L^2 - x^2}}, & |x| \le L, \\ 0, & |x| > L. \end{cases}$$

The validity of this transformation can be confirmed by comparing with the exact results for arbitrary Lévy index α

$$P_{st}(x) = \frac{(2L)^{1-\alpha} \Gamma(\alpha)}{\Gamma^2(\alpha/2)(L^2 - x^2)^{1-\alpha/2}}$$

S.I. Denisov, W. Horsthemke, P. Hänggi Phys. Rev. E 77, 061112 (2008).

Formula was derived by using the special conditions for impermeable boundaries at $x = \pm L$.



Spectral characteristics of steady-state Lévy flights in monostable confined potential



Stationary probability densities $P_{st}(x)$ for different values of the Lévy index α . The value of the parameters are: $\gamma = 1$, $D_{\alpha} = 1$ and L = 1.

Spectral characteristics of steady-state Lévy flights in monostable confined potential



Various curves correspond to different values of the exponent m of potential m.¹⁵

Spectral characteristics of steady-state Lévy flights in monostable confined potential



The spectral power density for m = 100 and for different values of the Lévy index α in log-log scale

$$S(\omega) \sim \frac{1}{\omega^{1+\nu}}, \qquad \omega \to \infty.$$

According to Newton's dynamics, the motion of a particle in a viscous medium in the potential profile U(x,y) under the action of random external forces in a 2D plane can be described by a system

To obtain a closed equation for the joint probability density function of random Markovian processes x(t) and y(t) which can be written in the form of the average

$$P(x, y, t) = \langle \delta(x - x(t))\delta(y - y(t)) \rangle$$

we apply the functional method developed in A. Dubkov, B. Spagnolo, FNL, 05, L267 (2005).

The general Kolmogorov equation for the joint PDF

$$\begin{split} \frac{\partial P(x,y,t)}{\partial t} &= \frac{\partial}{\partial x} \left(\frac{\partial U}{\partial x} P(x,y,t) \right) + \frac{\partial}{\partial y} \left(\frac{\partial U}{\partial y} P(x,y,t) \right) \\ &+ \int_{-\infty}^{+\infty} \frac{\rho_1(z)}{z^2} \left(e^{-z\frac{\partial}{\partial x}} - 1 + z\frac{\partial}{\partial x} \right) P(x,y,t) dz + \int_{-\infty}^{+\infty} \frac{\rho_2(z)}{z^2} \left(e^{-z\frac{\partial}{\partial y}} - 1 + z\frac{\partial}{\partial y} \right) P(x,y,t) dz. \end{split}$$

Further we find the steady-state joint probability distribution in the potential with radial symmetry in the case of two identical noises

$$U(x,y) = \gamma(x^2 + y^2)/2 = \gamma r^2/2.$$
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Gaussian white noise sources

$$\rho_1(z) = \rho_2(z) = 2D\delta(z)$$
 $P_{st}(x,y) = \frac{\gamma}{2\pi D} e^{-\frac{\gamma(x^2+y^2)}{2D}}$



2D-plot of steady-state joint PDF for the harmonic potential in the case of white Gaussian driving noises. The values of parameters are D = 5, γ = 2

Lévy noise sources



a) $D_1 = 0.5$ b) $D_1 = 5$

2D-plot of steady-state joint PDF for the harmonic potential subject to the Cauchy stable noises for different values D_1 ($\gamma = 2$)

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Stochastic approach to the description of memristors

A **memristor** is a non-linear passive two-terminal electrical component relating electric charge and magnetic flux linkage.



The memristor's electrical resistance is not constant but depends on the history of current that had previously flowed through the device, i.e., its present resistance depends on how much electric charge has flowed in what direction through it in the past; the device remembers its history — the so-called *non-volatility property.*



Schematics of I-V curve switching characteristics of memristors (CC – compliance current) with memory windows

PROBLEM:

a lack of stochastic models of memristors

TWO APPROACHES TO THE STUDY OF THE MEMRISTOR

- Discrete model (ideal Chua memristor with external Gaussian noise)
- Distributed model (stochastic macroscopic model based on FPE for dopants concentration)

For the ideal memristor the state-dependent Ohm's Law and its associated state equation are given by:

$$U(t) = R(q)I(t), \quad I(t) = \frac{dq}{dt}$$

Chua et al., IEEE Trans. Circuit Theory, 1971

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The ideal memristor can be defined by an equivalent algebraic function

$$w(t) = \int_0^t U(t)dt = \int_0^{q(t)} R(q)dq = \Phi(q(t)).$$

Firstly, we apply to the memristor a **stochastic voltage** U(t) in the form of a stationary **Gaussian noise** with non-zero mean U_0 and the correlation function $K(\tau)$.

According to the Central Limit Theorem, the process w(t) is again a Gaussian random process with the following probability distribution

$$P_w(y,t) = \frac{1}{\sqrt{4\pi D(t)}} \exp\left\{-\frac{(y-U_0t)^2}{4D(t)}\right\} \qquad D(t) = \int_0^t (t-\tau) K(\tau) d\tau.$$

Charge-controlled memristor

We apply the theorem of the probability theory to calculate the PDF of the charge flowing through a memristor

$$P_q(z,t) = \frac{\Phi'(z)}{\sqrt{4\pi D(t)}} \exp\left\{-\frac{\left[\Phi(z) - U_0 t\right]^2}{4D(t)}\right\}$$

Also we can obtain the PDF of the resistance using the same technology

$$P_R(r,t) = \frac{r}{\sqrt{4\pi D(t)}} \sum_k \frac{1}{|\Phi''(q_k(r))|} \times \exp\left\{-\frac{\left[\Phi(q_k(r)) - U_0 t\right]^2}{4D(t)}\right\} \qquad R = \Phi'(q)$$

We consider the following monotonic exponential dependence of the resistance on charge

$$R(q) = R_{ON} + \frac{\Delta R}{e^{-(q+q_1)/q_0} + 1}, \qquad \Delta R = R_{OFF} - R_{ON} \text{ and } R_{ON} \ll R_{OFF}$$

The exact expression for the PDF of the resistance has the following form

$$P_R(r,t) = \frac{q_o \Delta R}{\sqrt{4\pi D(t)}} \frac{r}{(R_{OFF} - r)(r - R_{ON})} \times \exp\left\{-\frac{\left[\Phi(q(r)) - U_0 t\right]^2}{4D(t)}\right\},\$$

where

$$\Phi(q(r)) = -q_1 R_{ON} - q_0 \Delta R \ln\left(e^{q_1/q_0} + 1\right) + q_0 R_{OFF} \ln\left(\frac{\Delta R}{R_{OFF} - r}\right) - q_0 R_{ON} \ln\left(\frac{\Delta R}{r - R_{ON}}\right)$$

$$K(\tau) = 2D\delta(\tau), \ D(t) = Dt$$



PDF of resistance in the case of white Gaussian noise U(t) for different time moments (a) $U_0 = 0$, (b) $U_0 = 1$.

The parameters are $q_0 = 1$, $q_1 = 0.1$, $R_{on} = 1$, $R_{off} = 5$, D = 0.5.

$$K(\tau) = \sigma^2 \exp\left(-t/\tau_c\right), \quad D(t) = \sigma^2 \tau_c \left[t - \tau_c \left(1 - e^{-t/\tau_c}\right)\right].$$



PDF of resistance in the case of colored Gaussian noise U(t) for different time moments (a) $U_0 = 0$, (b) $U_0 = 5$.

The parameters are $q_0 = 1$, $q_1 = 0.1$, $R_{on} = 1$, $R_{off} = 5$, $\sigma_2 = 1$, $\tau_c = 1$.

For comparison we apply to memristor **the current** I(t) **in the form of a stationary Gaussian noise** with zero mean and the correlation function $K(\tau)$.

The PDF of charge is a Gaussian process with the following probability distribution

$$P_q(z,t) = \frac{1}{\sqrt{4\pi D(t)}} \exp\left\{-\frac{z^2}{4D(t)}\right\}.$$

$$I(t) = \frac{dq}{dt}$$

The PDF of the memristance in this case has the following form

$$P_R(r,t) = \frac{1}{\sqrt{4\pi D(t)}} \sum_k \frac{1}{|R'(q_k(r))|} \exp\left\{-\frac{q_k^2(r)}{4D(t)}\right\}.$$

For considering case of an exponential dependence of the memristance on charge we obtain

$$P_R(r,t) = \frac{1}{\sqrt{4\pi D(t)}} \frac{q_0 \Delta R}{(R_{OFF} - r)(r - R_{ON})} \times \exp\left\{-\frac{1}{4D(t)} \left[q_1 + q_0 \ln \frac{R_{OFF} - r}{r - R_{ON}}\right]^2\right\}.$$

Current. White Gaussian noise



PDF of resistance for the case of white Gaussian noise excitation D(t) = Dtas a function of resistance and time.

The parameters are $q_0 = 1$, $q_1 = 0.1$, $R_{on} = 1$, $R_{off} = 5$, D = 0.5.

We consider a thin semiconductor film sandwiched between two metal contacts. The total resistance of the device is determined by two variable resistors connected in series

$$R_m = R_{ON}w(t) + R_{OFF}\left(1 - w(t)\right)$$

where w(t) = l(t)/L is the normalized size of the doped region ([0;1]); L is the full size of the memristor with two states: *Row* is the resistance of the memristor if it is completely doped (LRS) and *RoFF* is its resistance if it is undoped (HRS).

In this model the current I(t) and the voltage U(t) are connected by the following relation





We analyse the case of the applied current I(t) in the form of **white Gaussian noise** with nonzero mean and the intensity $2D_1$. The charge q(t) is again Gaussian process, but the probability distribution of the bounded random process w(t) is non-Gaussian and contains two deltafunctions

$$P_w(y,t) = p_1(t)\delta(y) + p_2(t)\delta(1-y) + \frac{1}{\sqrt{4\pi D_1 c_0^2 t}} \cdot \exp\left\{-\frac{(y-c_0 I_0 t)^2}{4D_1 c_0^2 t}\right\} \ 1_{(0,1)}(y),$$

$$p_1(t) = \int_{-\infty}^0 \frac{1}{\sqrt{4\pi D_1 c_0^2 t}} \exp\left\{-\frac{(y - c_0 I_0 t)^2}{4D_1 c_0^2 t}\right\} dy, \qquad c_0 = \mu_V R_{ON}/L^2$$
$$p_2(t) = \int_1^\infty \frac{1}{\sqrt{4\pi D_1 c_0^2 t}} \exp\left\{-\frac{(y - c_0 I_0 t)^2}{4D_1 c_0^2 t}\right\} dy.$$

Based on the same technique we calculate the PDF of the resistance as

$$P_R(r,t) = \frac{1}{\Delta R} \cdot P_w\left(\frac{R_{OFF} - r}{\Delta R}, t\right)$$

The Fokker–Planck equation (FPE) for the concentration of particles n(x,t)

$$\frac{\partial n}{\partial t} = \frac{1}{\eta} \frac{\partial}{\partial x} \left[\frac{\partial U(x, V)}{\partial x} n \right] + D \frac{\partial^2 n}{\partial x^2}, \qquad D = \theta / \eta^2$$

The potential profile U(x, V) for hopping particles is represented by the potential wells separated by the barriers (external field provides the periodic component $\Phi(x)$ and the slope F)

$$U(x, V) = \Phi(x) - Fx, \qquad F = qV/\varepsilon L$$

The Brownian diffusion in tilted periodic potential can be replaced by the diffusion in the flat tilted potential U_1 without barriers

$$U_1(x,V) = -v_{\rm eff}x,$$

As a result, FPE for the coarse-grained concentration of particles $n_1(x,t)$ takes the following form

$$\frac{\partial}{\partial t}n_1(x,t) = \frac{\partial}{\partial x} \left[n_1(x,t) \frac{\partial U_1(x,V)}{\partial x} \right] + D_{\rm eff} \frac{\partial^2}{\partial x^2} n_1(x,t)$$

and the exact expressions for the effective drift and diffusion coefficients, valid for arbitrary values of F and θ , are the following

$$v_{\rm eff} = \frac{\ell}{T_1(x_0, x_0 + \ell)}, \qquad D_{\rm eff} = \frac{\ell^2}{2} \frac{\Delta T_2(x_0, x_0 + \ell)}{\left[T_1(x_0, x_0 + \ell)\right]^3}$$

For modeling we used the following boundary conditions

 $n_1(0,t) = N_1, \ n_1(L,t) = N_2,$ (\diamondsuit)

where 0 and L are the coordinates of the TE and BE made of different materials.

Stationary solution of FPE reads as

$$n_{\rm st}(x) = \frac{N_2 - N_1}{\exp\left(\frac{v_{\rm eff}L}{D_{\rm eff}}\right) - 1} \left[\exp\left(\frac{v_{\rm eff}x}{D_{\rm eff}}\right) - 1\right] + N_1.$$

If BE is made of inert material with a very high work function, it can be modelled as a reflecting boundary, that is infinitely high barrier for the defects at the point x = L. Replacing boundary conditions (\diamondsuit) with the following ones

$$n_{\rm st}(0) = N_1, \ G_{\rm st}(L) = v_{\rm eff} n_{\rm st}(L) - D_{\rm eff} \frac{\mathrm{d}n_{\rm st}(x)}{\mathrm{d}x}\Big|_{x=L} = 0.$$

we get stationary solution of FPE

$$n_{\rm st}(x) = N_1 \exp\left(\frac{v_{\rm eff}x}{D_{\rm eff}}\right).$$



Stochastic model of memristor



I – V characteristic of the memristive device. Color lines: experimental, measured on the device based on Au/Ta/ZrO₂(Y)/Ta₂O₅/TiN/Ti structure (different colors correspond to different switching cycles). Black line: theoretical, based on numerical simulation with boundary conditions (◊).

$$\tau = \frac{2L}{v_{eff}} \frac{2D_{eff}/v_{eff}L}{1 + \pi^2 \left(2D_{eff}/v_{eff}L\right)^2}$$



Relaxation time as a function of dimensionless noise intensity θ/E_a for potential profile with equal widths of barriers and wells a = b, where E_a is activation energy at V = 0 (solid line). Inset: the same relaxation time as a function of dimensionless noise intensity but for large values of θ/E_a.

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Main results of Chapter 1

- The exact analytical results for the statistical characteristics of the residence time for anomalous diffusion in the form of Lévy flights in the inverse parabolic potential were obtained. NES phenomenon was observed in the system investigated.
- The exact analytical expression for the correlation time of Lévy flights in the symmetric bistable quartic potential has been first obtained. The correlation time ceases to depend on the height of potential barrier separating the two stable states for sufficiently high barriers unlike the Kramers' law for Brownian motion.
- The exact analytical result for the stationary PDF of the particle position in the asymmetric bistable quartic potential in the case of the unit Lévy index has been found.
- The new analytical expression of the steady-state PDF for Cauchy-Lévy flights in the symmetric steep potential well has been derived. In the limit m→∞ results coincide with those previously obtained for the infinity well of deep rectangular potential profile, without considering the problem of the boundary conditions.
- The asymptotic expression of the spectral power density for the steady-state superdiffusion in symmetric steep potential profiles, for arbitrary Lévy noise index α, has been found.
- For 2D diffusion the general Kolmogorov equation for the joint PDF of particle coordinates has been obtained by functional methods directly from two Langevin equations with statistically independent noise sources.

- Two models of an ideal Chua memristor with the external Gaussian noise have been investigated.
 - For the charge-controlled memristor the exact analytical expressions for the PDF of the memristance is found in general case. In the specific example of an exponential dependence of R(q) the influence of the noise mean value and the type of driven Gaussian noise on the memristors switchings between two states is analysed.
 - For the current-controlled memristor we obtain exact analytical expressions for the PDF of the memristance.

- We proposed a simple stochastic model for memristive systems which is validated experimentally and reproduces some fundamental properties of resistive switching.
 - The steady states of the model systems are shown to be of equilibrium or nonequilibrium depending on the boundary conditions, which in turn depend on the materials of the electrodes.
 - The relaxation time to the stationary state is obtained in analytic form and it has a nonmonotonic dependence on the intensity of the fluctuations for a certain set of values of the external parameters.
 - Some specific shapes of potential profiles, that describe the internal structure of the memristive material, are shown to accelerate the relaxation process. This paves the way to the use of noise as a control parameter for switching dynamics, and provides insight on the interplay between the properties of the dielectric structure and the switching times of the memristors.

THANK YOU FOR YOUR ATTENTION